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Analysis and Design of R.C. Deep Beams by Finite Strip Method and Comparison of Theoretical Results with Experimental Results

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***Abstract* – This paper describes analysis and design of deep beams subjected to two point loads with different L/D ratios using Program in FORTRAN 77 by finite strip method and standard codes (I.S. 456-2000, B.S.8112, ACI 318 and Appendix A of ACI 318). Variation of flexural stress, strain and shear stress in deep beam were plotted. The parameter, Shear span of beam was varied during the analysis. Several beams were cast and tested in laboratory as per codal provisions.**

***Keywords* : Deep Beam, Finite Strip method, Design**

I. Introduction

Beams with large depths in relation to spans are called deep beams. In IS-456 (2000) Clause 29, a simply supported beam is classified as deep when the ratio of its effective span L to overall depth D is less than 2. Continuous beams are considered as deep when the ratio L/D is less than 2.5. The effective span is defined as the centre-to-centre distance between the supports or 1.15 times the clear span whichever is less.

II. Objective of The Study

The main objective of this investigation is to conduct an experimental study on strength & behavior of deep beam. The detailed analysis was carried out using the finite strip method. The study also aimed at testing validity & usefulness of IS 456:2000, B.S.8112, ACI 318-2005 and ACI Appendix A (STM) code.

The objectives of the experimental investigation can be listed as follows,

1. To observe & explain the deflection, cracking & failure modes of deep beams subjected to two points loads;
2. To compare the flexural steel requirement as per codal provisions with that calculated using the finite strip method;
3. To comment on suitability of finite strip method & codal provisions.

III. Analysis of Deep Beam

A. Finite Strip Method

The finite strip approach was first introduced by CHEUNG (1968). For a structure with constant cross section and end boundary conditions that do not change transversely, stress analysis can be performed using finite strips. It is recognized as best method of analysis for simply supported rectangular plate, deep beam and box structure in terms of accuracy and efficiency. Basically, the method is a hybrid procedure which retains advantages of both the orthotropic plate Method and finite element concept.

B. The Computer Program

A computer program has been prepared for the analysis of Deep Beam having simple support. It was necessary for the solution of equations. It should be noted that the overall stiffness matrix is symmetrical. Computer program is developed on the basis of direct stiffness method.

The essential steps in writing a program were as follows,

1) Presenting input data to computer;
2) Evaluation of stiffness matrix of individual strips;
3) Assembling of structure stiffness matrix;
4) Forming the load vector;

5) Solving the assembled equations for the displacements;

6) Computing the internal forces in the members and reactive forces at the support;

7) Presentation of the results.

Features of the Program

1. The programming language used is FORTRAN77;

2. The program can handle any number of joints and members depending upon memory allocations available with PC;

3. The program can handle yielding of the support in all three directions. Also, it can handle symmetric structures in-plane, point load, loads, etc.

1. Variation of Flexural Strain

The parametric study to know strain distribution in case of deep beam was performed. It is found that the smaller the span/depth ratio (i.e., less than 2.0), the more pronounced the deviation of the strain pattern from that of Euler Bernoulli theory. Figure 1 & Figure 2 shows that the flexural strain at mid span of simply supported deep beam for two different shear span–to-depth ratios. The beams have disturbed region in flexural strain distribution. Deep beams behave differently from shallow beams. In these members, the distribution of strain across the depth of the cross section is nonlinear and a significant amount of load is carried to the supports by a compression strut joining the load and the reaction. These structural elements belong to D (disturbed) regions. Structural members can be broadly divided into two regions, namely, B (or Bernoulli) regions where the strain distributions are linear, and D (or Disturbed) regions where the strain distributions are non–linear. While well defined theories are available for designing B regions, thumb rule or empirical equations are still being used to design D regions, though B and D regions are equally important. Schlaich *et al.* (1987) identified deep beams as discontinuity regions where the strain distribution is significantly nonlinear and specific strut-and-tie models need to be developed, whereas shallow beams are characterized by linear strain distribution and most of the applied load is transferred through a fairly uniform diagonal compression field.

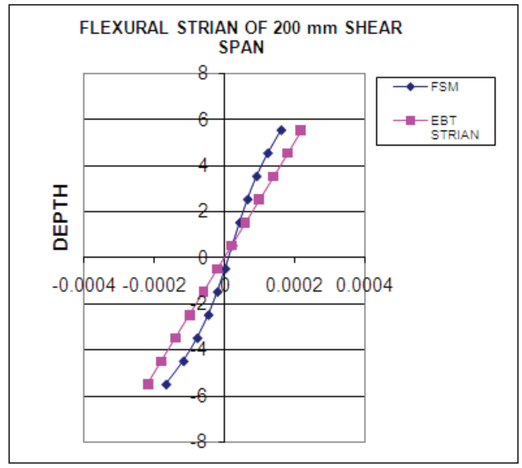

Fig.1 Flexural Strain Distribution shear span-to-depth ratio 0.57

From the variation of flexural strain graphs the definition of simply supported deep beam as per IS 456:2000 i.e. L/D ratio is less than or equal to 2.0 is reasonably accurate.

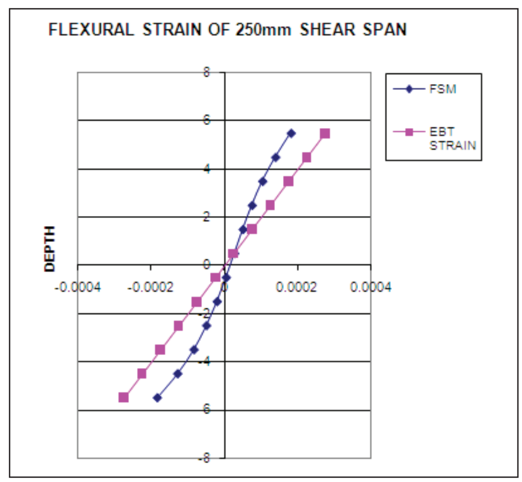

Fig. 2 Flexural Strain Distribution shear span-to-depth ratio 0.71

2. Variation of Flexural Stress

The stresses in isotropic homogeneous deep beams can be determined using finite strip analysis. It is found that the smaller the span/depth ratio (i.e., less than 2.0), the more pronounced the deviation of the stress pattern from that of Euler Bernoulli theory. Figure 3 & Figure 4 shows the flexural stress at mid span of simply supported deep beam for two different shear span–to-depth ratios. The tensile stresses increase rapidly at the bottom and neutral axis moves towards soffit of the beam.

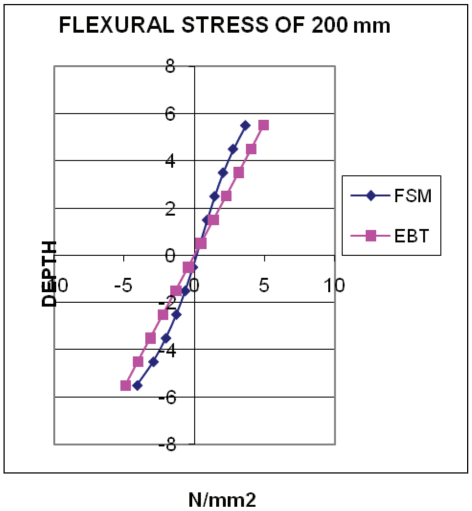

Fig. 3 Flexural Stress Distribution shear span-to-depth ratio 0.57

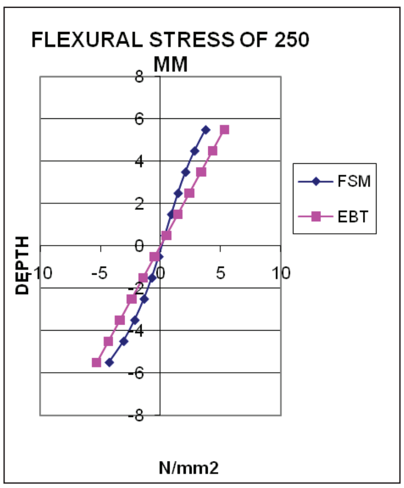

Fig.4 Flexural Stress Distribution shear span-to-depth ratio 0.71

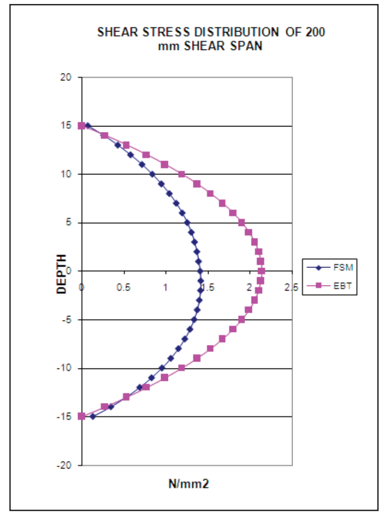

Fig. 5 Shear Stress Distribution shear span-to-depth ratio 0.57

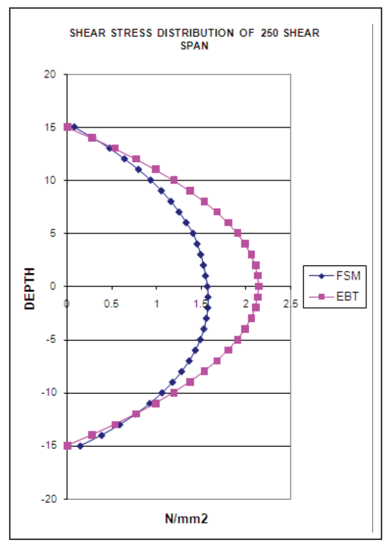

Fig. 6 Shear Stress Distribution shear span-to-depth ratio 0.71

From the variation of flexural stress graphs the definition of simply supported deep beam as per IS 456:2000 i.e. when L/D ratio is less than or equal to 2.0 is reasonably accurate

3. Variation of Shear Stress

Figure 5 & Figure 6 shows the shear stress near support of simply supported deep beam for two different shear span –to-depth ratios .The beams have drastic change in shear stress distribution. Deep beams behave differently from shallow beams. The shear stress patterns have also changed in case of deep beam. It is found that the smaller the span/depth ratio (i.e., less than2.0), the more pronounced the deviation of the shear stress distribution from that of Euler Bernoulli theory.

From the variation of shear stress graph it is clear that shear effect is predominant in beams having L/D ratio less than or equal to 2.0 which may lead to warping of the section.

IV. Tension Reinforcement Calculations From Graph

Tension Reinforcement required was calculated from the flexural stress graphs which are plotted by using Finite Strip program.

Reinforcement required for Shear span 200 mm:

Sample calculation of reinforcement for bottom most strip

Area of steel required

$$= \frac{\text{Flexural stress in strip x Area of Strip}}{\text{Design stress in steel}}$$

$$= \frac{\sigma_y \times A_{strip}}{0.87\, f_y}$$

$$= \frac{4.10687 \times 31.819 \times 150}{0.87 \times 415}$$

$= 54.29\ mm^2$

Similarly calculations for all the strips are done and tabulated.

TABLE I REINFORCEMENT REQUIRED AS PER FSM FOR SHEAR SPAN 200 MM

Sl. No.	Strip No.	Reinforcement required mm^2
1	1	0.86
2	2	8.99
3	3	17.3
4	4	27.183
5	5	38.973
6	6	54.293
Total		147.6 mm^2

Reinforcement required for Shear span 250 mm:

Sample calculation of reinforcement for bottom most strip

Area of steel required

$$= \frac{\text{Flexural stress in strip x Area of strip}}{\text{Design stress in steel}}$$

$$= \frac{\sigma y \times A_{strip}}{0.87\, f_y}$$

$$= \frac{4.58062 \times 31.819 \times 150}{0.87 \times 415}$$

$= 60.55\ mm^2$

Similarly calculations for all the strips are done and tabulated.

TABLE II REINFORCEMENT REQUIRED AS PER FSM FOR SHEAR SPAN 250 MM

Sl. No.	Strip No.	Reinforcement required mm^2
1	1	0.96
2	2	10.032
3	3	19.296
4	4	30.32
5	5	43.805
6	6	60.55
Total		164.97 mm^2

V. DESIGN OF DEEP BEAMS

A. Introduction

Deep beams were designed for two point loads and for two shear spans viz. 200 mm and 250 mm. Point loads of 50 kN are applied on deep beams. Dimensions of deep beams chosen for design purpose are,

Length = 700 mm,

Depth = 350 mm,

Thickness = 150 mm

B. Design Methods

Design of deep beams was done by following methods.,

1. Design by using I.S.456-2000 method;
2. Design by using B.S.8112 method;;
3. Design by using ACI-318 method;
4. Design by using ACI-Appendix a (Strut & Tie) method;

For each method mentioned above, several- beams with 200 mm and 250 mm shear span were designed and cast for experimental study.

VI. EXPERIMENTAL WORK

Deep beams are designed by using I.S.456-2000, B.S.8112, ACI-318 and ACI-Appendix A (strut & Tie method) for two points loading and for several shear spans. Dimensions of Deep beams chosen for design purpose are,

Length = 700 mm,

Depth = 350 mm,

Thickness = 150 mm

VII. Testing in Laboratory and Test Results

Fig. 7 Deep beam testing

Mode of failure was found to be shear with diagonal tension & can be categorized as given in table.

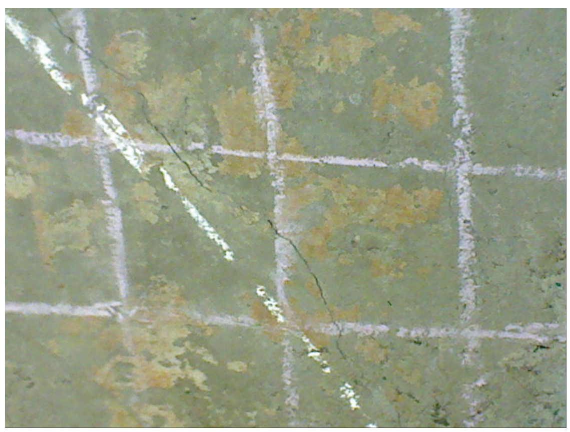

Fig. 8 Diagonal cracking in deep beam

Fig. 9 Strut formation in deep beams

TABLE III SAMPLE TEST RESULTS

Loading: Two point loading, each point load of 50 kN (working load)

Beam dimensions: Total Length = 700 mm, Effective Span = 600 mm,

Depth = 350 mm, Thickness = 150 mm, Average cube strength = 21 N/mm^2

Beam No.		B 1/1	B 1/2	B 2/1	B 2/2	B 3/1	B3/2	B 4/1	B 4/2
Design Method		I.S.456	I.S.456	B.S.8112	B.S.8112	ACI 318	ACI 318	Strut & Tie	Strut & Tie
Shear span (mm)		200	250	200	250	200	250	200	250
Shear span to depth ratio		0.57	0.71	0.57	0.71	0.57	0.71	0.57	0.71
Reinforcement provided (No.of bars)	Flexure steel Required in mm^2	160.74	199.845	160.74	199.85	231.33	231.33	169.52	215.89
	Flexure steel Provided i) 10 mm Φ ii) 08 mm Φ iii) mm^2	2 1 207.24	2 1 207.24	2 1 207.24	2 1 207.24	2 1 235.62	2 1 235.62	- 4 200.96	3 - 235.62
	Shear Required in mm^2 Vertical Horizontal	126 105	126 105	113.04 84.78	113.04 84.78	282.6 113.04	282.6 113.04	262.5 72	262.5 72
	6 mm dia. Vertical Horizontal	6 2	6 2	4 3	4 3	9 4	9 4	5 3	5 3
Load at first crack	Total	200kN	190kN	180kN	170kN	220kN	210kN	210kN	200kN
	Each Point load	100kN	95kN	90kN	85kN	110kN	105kN	105kN	100kN
Failure Load	Total	300kN	280kN	285kN	275kN	340kN	334kN	330kN	310kN
	Each Point load	150kN	140kN	142.5kN	137.5Kn	170kN	167kN	165kN	155kN
Deflection at failure	Total	3.4 mm	3.8 mm	3.5 mm	4 mm	3.6 mm	3.75 mm	3.5 mm	3.7 mm
	Permissible deflection	2.4 mm	2.4 mm	2.4 mm	2.4 mm	2.4 mm	2.4 mm	2.4 mm	2.4 mm
	Deflection at 150 kN load	1.03 mm	1.37 mm	1.24 mm	0.9 mm	1.10 mm	1.26 mm	1.33 mm	1.52 mm
Observed mode of failure		Mode II- 3	Mode II- 3	Mode II- 3	Mode II- 3	Mode II- 3	Mode II- 3	Mode II- 3	Mode II- 3

Referring to table nos. I, II and III, it is found that flexural steel reinforcement as per FINITE STRIP METHOD is less than that specified by codes. Description of modes of failure as described by Salamy *et al* [8]:

Failure modes of deep beam can be divided in following two main categories,

a. Flexural failure mode

b. Shear failure mode

Shear failure mode can be sub divided into following three categories,

Mode II-1: Diagonal tension failure, which in the line of thrust become so eccentric and give rise to flexural failure in compressive zone. It is important however to mention that this kind of failure is a result of tensile crack extension in compressive zone due to flexural load.

Mode II-2: Shear compression failure where RC beam fails due to the development of diagonal crack into the compressive zone and reduces the area of resisting region excessively and beam crushes once generated compressive stress exceeds compressive strength of concrete.

Mode II-3: Shear proper or compressive failure of struts, which is often observed in beams with very small shear span to depth ratio (a/d < 1.5). In this case due to the small a/d ratio, the line of thrust will be so steep and arch action not only reserve flexural capacity in most cases but also efficiently sustains required shear force. Arch is clearly observed in those beams and finally beams fail due to either sudden tensile crack formation parallel to the strut axes or compressive crush in normal direction to the strutaxes.

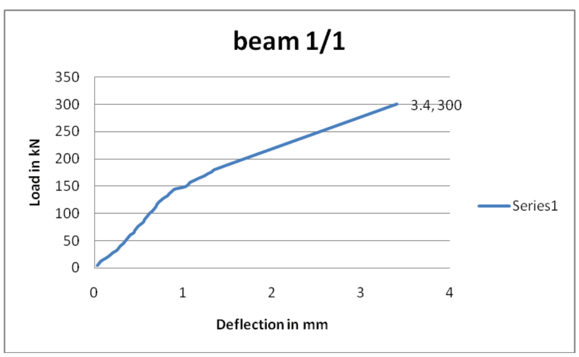

Fig. 10 Graphs of Load VS Deflection

VIII. Conclusion

Following conclusions were drawn from above studies,

1. Failure of deep beams was mainly due to diagonal cracking and it was along the lines joining the loading points and supports;
2. The strength of beams with 250 mm shear span is about 5 % less than that of 200 mm shear span. It is clear from these results that the strength of deep beam is inversely proportional to the shear span for the constant depth of the beam;
3. It was observed that the arching action of the main tension steel & the web steel together with concrete will carry the shear ;
4. All the beams fail due to shear hence deflection at failure was low;
5. The overall average load at first crack was found approximately half of the ultimate failure load. Therefore in design of deep beams, a load factor of 1.5 seems to be reasonable;
6. The flexural steel requirement of IS456:2000 & BS 8112-2006 methods are more by a margin of 8.17 % than Finite Strip Method. In case of STM method this margin was found to be 12.93 %. But in case of ACI-318 method it was 36.19 % more than FSM method. Therefore it can be concluded that tensile reinforcement requirements of I.S., B.S. & STM methods are near to the FSM whereas the same by ACI -318 methods is more. Therefore the strength of beams designed by ACI -318 method was about 10 % more than other beams;
7. Web steel requirement of ACI-318 method is more than other methods due to specification of minimum spacing of d/5. Due to more web steel, initial cracking load of the beams designed by ACI-318 method is about 7 % more than that of the beams designed by other methods. Even if web reinforcement does not contribute substantially to the strength of deep beams, it prevents initial cracking of beam at low loads;
8. The flexural tensile force as per the FSM analysis is concentrated in lower 1/3 height for all the beams. Therefore in the deep beams loaded with two point loading, steel for the flexural tensile force may be provided mainly in this height. This is matching with all the codal provisions.

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